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Inequality Measures, Equivalence Scales and Adjustment for Household Size and Composition

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Abstract

Total household income inequality can be very different from inequality measured at the income per-capita level but only in recent years has the pattern of this divergence been investigated. In this paper, results from Coulter et al. (1992) using a one-parameter equivalence scale are updated using data for Ireland, Italy, the UK and the US. A class of two-parameter equivalence scales, representing relative weights of adults and children, is then analysed. Results are shown to depend on the correlation between household size, household composition and household income. Inequality generally increases with children's weight and decreases with adults' weight. OECD and other two-parameter equivalence scales empirically used show a similarity of results to one-parameter equivalence scales with elasticity around 0.5.

1. Introduction

Comparative studies of income distribution are heavily affected by the "technical" parameters chosen by researchers in measuring inequality. One of the major issues here involved is the assessment of the extent and the change in inequality when different adjustments for household size and composition are allowed.[1] Up to recent years, the relevant unit in inequality studies was chosen between household income (H), household income per-capita (Y) and individual income. While individual income is the simplest unit of analysis, a better alternative is to consider H, as the household is the locus of decisions on income getting and income spending of individual members. A preferable solution is represented by Y, in which total household income is adjusted by the household size and total income is shared equally among all the household members.

However, the measurement of Y rules out the possibility to attach different economies of scale to households of different size: in other words, the underlying assumption of the household income per-capita analysis is that the well-being of an individual sharing £20,000 in a two-person household is the same as the well-being of an individual sharing £40,000 in a four-person household. It seems more reasonable to postulate the existence of positive economies of scale within larger households; hence, a consistent measure of individual well-being W can be represented in (1):

$$W = H / S^{\varepsilon} \qquad \textbf{(1)}$$

[1] Other preliminary steps refer to i) the modification of extreme incomes (procedure known as bottom and top recoding); ii) the definition of concepts such as gross income or disposable income, which generally vary across countries; iii) the several alternative definitions used in national household surveys: families (not households) are observed in Italy while tax units are considered in Switzerland. Also the definition of household varies in some complicated ways, as it is the case of Norway. For a more exhaustive description of these issues, see Atkinson et al. (1995); and iv) the indices used to assess inequality (indices are neither ordinally nor cardinally equivalent). On this last point, see Cowell (1995). See also Champernowne (1974), Figini (1998), Bigsten (1991) and Sundrum (1990). On the technical aspects of managing household surveys a good overview is provided in Atkinson et al. (1995).

where H is the sum of individual incomes in the household (total household income), S is the household size and ε is a parameter representing the economies of scale. ε ranges from 0 (perfect economies of scale) to 1 (no economies of scale). Therefore, household income (ε = 0) and household income per capita (ε = 1) are the two extreme cases of a welfare analysis in which the elasticity of scale ε plays a fundamental role.[2]

Buhmann et al. (1988) find that all the equivalence scales empirically used can be approximated by a single parameter scale as (1) and in recent years one-parameter scales have been directly used (Atkinson et al., 1995 measure inequality in OECD countries considering ε = 0.5). This evolution raises a few questions about the "best" ε to use and about the pattern of inequality change when ε varies. While the former problem invokes thinking about welfare assumptions and economies of scale within households, the latter issue has been tackled theoretically and empirically by Coulter et al. (1992): by increasing ε from 0 to 1, inequality first decreases and then increases, thus depicting a U-shape. These general findings are re-assessed in the present paper.

Yet, equation (1) is a simplification of a more general formula in which other household characteristics such as composition, location and age might be considered. This approach in adjusting for household characteristics is represented in (2):

$$W = \frac{H}{\left(\alpha_1 N_1 + \alpha_2 N_2 + \ldots + \alpha_k N_k\right)^{\varepsilon}} \qquad \textbf{(2)}$$

where N_i is the size of each type *k* of components of the household (elderly people, adults, children…), α_ι is the relative weight given to them and ε

[2] Actually, household income inequality is not technically equal to equation 1 with a parameter ε = 0 because of the different weighting procedure applied to the data: in the former case we weight according to the number of households, in the latter to the number of individuals.

represents the economies of scale within the household. A particular sub-class of this formula will be analysed throughout the paper.

This paper is organised as follows: in the next section the Luxembourg Income Study (LIS) database, used in this work, is presented. Section 3 follows the procedure of Coulter et al. (1992) comparing inequality measured according to (1) in four different countries: Ireland, Italy, the UK and the US. In Section 4, a particular subclass of formula (2) is considered: a two-parameter equivalence scale which distinguishes between the household head, other adults and children in the household. A weight of 1 for the household head and weights α_1 and α_2 which range between 0 and 1 for other adults (N_1) and children (N_2) in the household are respectively used (3).[3]

$$W = \frac{H}{1 + \alpha_1 N_1 + \alpha_2 N_2} \qquad \textbf{(3)}$$

Section 4 provides also a comparison between the different scales used while Section 5 concludes.

2. The data

Since the Luxembourg Income Study (LIS) project was founded in 1983, a huge step towards a better understanding of inequality and its measurement has been taken. The project has four main goals: i) to create a database containing social and economic data collected in household surveys from different countries; ii) to provide a method allowing researchers to use the data under restrictions required by the countries providing the data; iii) to create a system to allow remote access and to elaborate data using computer networking and iv) to promote comparative

[3] OECD scale is a particular case of equation (3), in which α_1 = 0.7 and α_2 = 0.5. Other scales often used, attach values of 0.6 or 0.5 to α_1 and 0.4 or 0.3 to α_2.

studies on income aggregates. At this stage the LIS database includes about 70 datasets for 25 countries, covering the period from 1967 to 1995. Most datasets include three different files, the first with data at the household level (allowing sometimes also a disaggregation among multifamily households), the second at the individual level and the third at the child level. One of the main issues in setting up such a database is to elaborate data from single national household budget surveys transforming variables and re-weighting single cases in order to allow a satisfactory international comparison. Of course, perfect comparability will never be reached but the LIS database allows a good degree of comparability between several countries.[4]

3. Income Inequality and economies of Scale

How much does inequality change moving from household income to household income per-capita inequality? Is the change similar for all the indices? Is the change similar for all the countries?

To start with, the problem can be represented in the following way: household income per-capita (Y) is the ratio of household income (H) over household size (S).

$$Y = H / S \tag{4}$$

Considering logarithmic values and the coefficient of variation (CV) as a measure of inequality, Sundrum (1990) shows that $CV_Y < CV_H$ if:

$$2rCV_H CV_S > CV_S^2 \text{ or if } r > \frac{CV_S}{2CV_H} \tag{5}$$

[4] Technically, the preliminary stage of the research is to set up some jobs (using SPSS or SAS commands) which are sent via e-mail to the server address in the LIS headquarters in Luxembourg. These jobs are automatically executed and the output file is sent back to the original e-mail address in a few minutes. A complete documentation with description, frequencies and labels for each variable of the database is also available online (http://lissy.ceps.lu/) to allow researchers to overcome problems of definition and transformation that their own work can require.

where ρ is the coefficient of correlation between household size and household income. Since CV_S is usually smaller than CV_H, the right-hand side of equation (5) is sufficiently small compared to ρ if there is strong positive correlation between size and total household income (as there is in household data).[5] Therefore, in the generality of cases, Y inequality would be lower than H inequality. LIS database highlights this decrease in inequality moving from the household level to the household per-capita level, as Table 1 shows. The 22 countries for which information on inequality in the period 1987-1992 is available are listed. For each country, three different measures of inequality (Gini, Theil and Atkinson with parameter equal to 0.5) are computed for Household income (H) and Household income per-capita (Y).[6] Table 1 shows that inequality generally decreases moving from H to Y. The only cases for which inequality increases are Israel, Italy, Poland and US (only for Gini and Theil).

However, this change in inequality is not linear and can be fully understood once the possibility of introducing equivalence scales is allowed. This process adjusts household income with respect to household size in order to rule out the assumption of not having economies of scale within the household. If two households, one composed of two individuals with a total income of £20,000 and another one of four components and £40,000 are considered equivalent, as equation (4) assumes, the possibility of having intra-household economies of scale is ruled out. This assumption is now relaxed and households are adjusted in order to catch positive economies of scale within larger households, as the Buhmann scale (Buhmann et al., 1988) recalled in (1) represents.

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[5] The average household size is at the minimum in the bottom decile and at the maximum in the top decile for each country in the LIS database. The number of children increases up to the 3th-7th decile and decreases thereafter and, as a result, also the number of economic active persons (computed as the number of household components minus children) increases with total household income. Data for all the countries are available from the author. Data for Ireland, Italy, UK and US are published in Table 5.

[6] Inequality decreases also moving from household inequality to Economic Active Person (EAP) inequality, due to the same reason: EAP increase along the distribution of income for almost every country. EAP distributions and inequality values are available from the author.

Coulter et al. (1992), explain the theoretical relationship between equivalence scales and inequality; in analytical terms they refine the model above illustrated in (4) reducing it to a particular case of the general case in which well-being W_i of an individual is a function of four different variables, total household income (H), household size (S), elasticity of scale (ε) and household characteristics (η):

$$W_i = W(H_i, S_i, \varepsilon_\iota, \eta_\iota) \qquad \textbf{(6)}$$

This formula reduces to (4) if household characteristics (such as location, age, health) are normalised and the elasticity of scale is set equal to 1. In this section the analysis is broadened by allowing economies of scale to vary according to the Buhmann scale. The parameter ε represents the intensity of economies of scale and can range from 0 to 1. When ε equals 0, W reduces to H (perfect economies of scale are assumed); when ε is equal to 1, W reduces to Y as in (4) (economies of scale are ruled out and the well being of each individual is simply equal to the household income per-capita). Neither of these two cases are realistic because in each household there are some relatively fixed expenditures that are shared among its components (rent, bills) and the extent of the sharing mainly depends on the household size. Recalling the previous example, if there are economies of scale, an individual who shares £20,000 within a two-people household is worse off than an individual who shares £40,000 in a four-people household.

On the other hand, throughout the paper we will keep assuming that there is no intra-household inequality: H is postulated to be evenly distributed among the components of the household but this is not always true, particularly in the case of multi-family households.

Buhmann et al. (1988) and Coulter et al. (1992) demonstrate that the movement between household income and household income per-capita inequality is not linear but involves a U-shape with respect to ε. Inequality first decreases moving from ε = 1 to a lower value and, from a certain stage down to 0, inequality increases. When households are ranked according to their total income, rich households are the largest ones. Therefore, when income is adjusted via a parameter ε, the ratio between income and size in equation (1) decreases respectively more for rich individuals than for poor individuals, thus having an equalising effect on the distribution. But, for high values of ε, the re-ranking process acts to counter-balance this change in inequality: by increasing ε, the possibility to re-rank units in the distribution augments. The total effect on income inequality would depend on the values of the two effects. For low values of ε, the re-ranking effect is not sufficiently strong to reverse the equalising effect but, for a higher ε, the re-ranking will be sufficiently strong to lead an increase in inequality. This process can be understood by looking at the example outlined in Table 2.

Using variance as a measure of inequality, the introduction of the parameter ε can be represented as follows:

$$\text{VAR (w)} = \text{VAR (h)} + \text{VAR }(s^{\varepsilon}) - 2\rho\varepsilon\,(\text{VAR (h) VAR (s)})^{0.5} \qquad \textbf{(7)}$$

An increase in ε widens the gap between H and W inequality. But after a threshold level, the rise in ε implies a more likely re-ranking of the households causing an overall decrease in the gap. The result is a composite effect depicted by a U-pattern of inequality with respect to ε.

The above pattern has been tested by Cowell et al. (1992) on different indices of the General Entropy Measures family (GEM), including Theil, Atkinson and the Coefficient of Variation, and on the Gini index: data from the UK confirm the U-shape in inequality with respect to ε. They also find a different skewness of the U curve for different indices: keeping anything else constant, indices more sensitive to inequality among high-incomes (such as the Coefficient of Variation) show a U curve skewed to the left, more similar to a J-shape. Indices more sensitive to inequality among low-incomes (as Atkinson) show a U curve more skewed to the right, more similar to an inverted J-curve (for the explanation, see Coulter et al., 1992, pp 1073).

In this paper, LIS data for the UK 1991, the US 1991, Ireland 1987 and Italy 1991 are used. For each country Gini, CV, Theil and Atkinson (with parameter $\phi = 0.5$) indices are computed for both person and household weighting.[7] The main results are listed as follows and recalled in figures 1-2 (UK 1991), 3-4 (US 1991), 5-6 (Ireland 1987) and 7-8 (Italy 1991).

i) The U shape holds for all the countries and all the indices: inequality is a U-shaped function of ε.

ii) Contrary to the remark of Coulter et al. (Coulter et al., 1992, p.1077) the choice of whether to weight according to the number of individuals (PP curves in the figures) or households (HH curves in Figures 1 to 8) affects the robustness of results. Coulter et al. use household weights, finding that the McClements equivalence scale used by the British Institute for Fiscal Studies ($\varepsilon \cong 0.6$) actually minimises the extent of inequality. Using person weights, which are more appropriate in measuring well-being, the minimum of inequality is generally found to be related to a lower ε. Therefore, the U-curve is more skewed to the left than the curve drawn using household weights, thus taking more the shape of an inverted-J curve.

iii) The gap between inequality measured using PP and HH weights depends on ε. For some countries, namely the UK, the US and Ireland, the difference between PP and HH is minimised when $\varepsilon = 1$, but also alternative patterns appear: for Italy the gap diminishes up to the point of minimum inequality ($\varepsilon = 0.5$) and then goes up again.

iv) The minimum inequality values are represented in Table 3. Minima using PP weights are generally found in correspondence of a lower value of ε than HH

[7] To have results that are representative of the whole population, single cases from the sample have to be weighted. When H income ($\varepsilon = 0$) is measured, it seems appropriate to weight according to the number of households (HH). With other values of the parameter, since individual well-being, is analysed, it seems more appropriate to weight according to the number of individuals (PP). Here both possibilities are considered.

minima. The minimum for Gini and Atkinson indices corresponds to a higher ε (around 0.5/0.6 for HH); ε minimises inequality measured with Theil for a value around 0.4/0.5 while CV is minimised using ε around 0.4. This confirms the theoretical discussion by Coulter et al.: curves for indices which are particularly sensitive to high income inequality are more skewed to the left than indices sensitive to low income inequality. By using PP weights, the minimum corresponds to an ε around 0.5 for Atkinson and Gini and around 0.3 for CV. The shape of the curve also depends on the country; the US and Italy have minima around 0.3-0.5 while the UK and Ireland have minima in correspondence of ε around 0.4-0.6. Countries with higher inequality in household size distribution (UK and Ireland) seem to be more likely to have a U-shape skewed to the right than countries with lower inequality in household size distribution (U.S. and Italy, which seem to have a U-shape more skewed to the left. In Table 5, a measure of inequality (Coefficient of Variation) for the distribution of household size is represented in the last row. Ireland and the UK have a Coefficient of Variation of 0.175 and 0.186 respectively which is much higher than the value for Italy and the US which are 0.126 and 0.132.

In conclusion, the use of a particular parameter of elasticity is fundamental in determining not only the absolute level of inequality but also the ranking between countries. In fact, when the level of inequality between two or more countries is particularly close, as it is in the case of Ireland, the UK and the US, their ranking can be affected. In Table 4 is shown that the ranking of these three countries, given the most usual assumptions (PP weights and ε = 0.5) and using the Gini coefficient, is different from the ranking using different assumptions (HH weights and ε = 1), as in the third column of the same table. Another picture of the same situation, using the Atkinson index with person weights, is represented in Figure 9. The UK starts as the most unequal distribution for ε = 0 and becomes the most equal for ε = 1. Whether these changes are only marginal or have a substantial effect on the way in which inequality is perceived, it is a matter of personal judgement. Nevertheless, the use of a particular ε is a central issue in the determination of a country's level of inequality.

4. An analysis with a two-parameter equivalence scale

A more precise way to adjust for household characteristics is to measure individual welfare not only with respect to income and size, but also with respect to the number of earners, children and elderly people within the household. A general formula for this second approach is represented by equation (2). An example of such an equivalence scale is the OECD scale (8):

$$W = H / (1 + 0.7(N_{\alpha}-1) + 0.5N_c) \quad \textbf{(8)}$$

where N_{α} and N_c are the number of adults and children respectively and a weight of 1 is attached to the household head. Using the same procedure followed for the previous study of the one-parameter equivalence scale, the fundamental question that will be tackled in this section is: how do measures of inequality change when parameters α_1 and α_2 vary between 0 and 1?

Considering the variance as a measure of inequality, and the variables in logarithmic terms, we have that:

$$VAR(w) = VAR(h) + VAR\log(1+a_1S_1+a_2S_2) - 2COV(h, \log(1+a_1S_1+a_2S_2)) \quad \textbf{(9)}$$

with the movement of VAR(w) depending on the distributions of adults and children and their correlation with household income. To be more precise, the class of Generalised Entropy Measures (I) is considered:

$$I = \frac{1}{q(q-1)N}\left[\sum_i\left(\frac{W_i}{\overline{\overline{W}}}\right)^q - 1\right] \quad \textbf{(10)}$$

where θ is a parameter determining a particular aversion to inequality, N is the number of observations, W_i is the well being of an individual belonging to the i-th household and $\overline{W}$ is the average of the measure.[8] W_i is calculated according to (11):

$$W_i = \frac{H_i}{1 + \alpha_1 S_1^i + \alpha_2 S_2^i} \qquad \textbf{(11)}$$

where H_i is total income of the i-th household, S_1 is the number of adults in the household minus the head, S_2 is the number of children in the household, the weight of the household head is set equal to 1, and α_1 and α_2 are the weights of, respectively, other adults and children.

The differentiation of GEM with respect to changes in α_1 is shown in (12). A similar formula holds for changes in α_2, the only difference being the substitution of S_1^i and S_1^j with S_2^i and S_2^j in the numerator of M.

$$\frac{\partial I}{\partial \alpha_1} = \frac{1}{(\theta - 1)N} \sum_i \left(\frac{W_i}{\overline{W}}\right)^{\theta - 1} M \qquad \textbf{(12)}$$

where M is:

$$M = \frac{\dfrac{\overline{W} S_1^i W_i}{1 + \alpha_1 S_1^i + \alpha_2 S_2^i} - \dfrac{W_i}{N} \sum_j \dfrac{Y_j S_1^j}{\left(1 + \alpha_1 S_1^j + \alpha_2 S_2^j\right)^2}}{\overline{W}^2} \qquad \textbf{(13)}$$

Equation 13 can be read in this way. The change in inequality depends on θ and on the sign of M which, in turn, depends on the values of S_1 and S_2 in each household. Theoretically any sign can result and empirically this would depend

[8] When θ = 1, GEM is equivalent to the Theil index; when θ = 2 an index cardinally equivalent to the Herfindal index is obtained, when θ = 3 the index is ordinally equivalent to the Coefficient of Variation and when θ = 1 - ϕ, GEM is ordinally equivalent to the class of Atkinson indices for parameters equal to ϕ. See Cowell (1995) and Figini (1998).

on the type of distribution of adults and children among households. Table 5 shows that the number of adults generally increases along the distribution of total household income. Given a certain weight α_2, an increase in the weight given to the number of adults raises the denominator of equation (11), thus implying an equalising effects when rich and poor households are compared. Table 5 also shows that the distribution of children is more heterogeneous. In the four countries under consideration, the number of children per household increases up to the 3rd decile (Ireland and Italy), to the 5th (US) and to the 7th (UK), declining thereafter. Given α_1, an increase in the weight given to children has instead a disequalising effect because rich households have, generally, less children. The total effect, increasing at the same time α_1 and α_2, is less clear and also depends on the re-ranking effect, the absolute values of S_1 and S_2 and the value of parameter θ in GEM.

Empirically, the complexity of the situation and the possibility to have contrasting results is fortunately reduced. Given the similar pattern in adults and children distributions among countries, a few stylised facts can be highlighted (see also figures from 10 to 14).

i) When α_2 is hold fixed, inequality decreases with increases in α_1 (See Figure 10b); for low values of α_2, however, an inverted J-shape can appears with inequality increasing at high values of α_1 (see Figure 10a).

ii) When α_1 is hold fixed, inequality increases with α_2 (see Figure 11a); for high values of α_1, however, a J-shape can appear with inequality decreasing at low values of α_2 (see Figure 11b). Again, these non monotonic patterns do not appear in every country but only in countries in which household size inequality is higher (Ireland and the US).

iii) When the two weights vary together, the overall trend is depicted by an inclined surface with highest inequality for low values of α_1 and high values of α_2 and lowest inequality for high values of α_1 and low values of α_2. Increasing both weights together we obtain again a U-shape (see Figures 12-15) but also a monotonic increase in inequality can appear (see Figure 16).[9] Inequality is more sensitive to changes in children weights than to changes in adults weights.

iv) Particular scales have remarkable importance: the OECD scale (α_1 = 0.7 and α_2 = 0.5) and other two scales in which weights are α_1 = 0.6 and α_2 = 0.4 and α_1 = 0.5 and α_2 = 0.3 respectively. Among two-parameter scales, their values are in the middle of the range of possible results, with the OECD scale showing the highest measure among the three. Compared to one-parameter scales, Table 6 shows that they are very close to the value that we would get using ε around 0.5-0.6.

5. Concluding remarks about inequality comparisons

While the few regularities found in the previous section still need further statistical analysis, it is evident that the decision on the equivalence scale and the parameter to use in inequality measurement is fundamental. A few conclusions about the effects of this choice can be outlined as follows:

i) The absolute measure of inequality is heavily dependent on the type of adjustment for household size and composition and on the value of the parameter describing economies of scale within the household.

ii) Among one-parameter scales, each country and each index has a peculiar way to react to the choice of the elasticity of scale. While the U-shape depicted when rising the value of ε holds in the generality of cases, the skewness of the curve, the difference between two alternative weighting procedures (PP and HH weights) and the value of ε for which inequality is minimised vary considerably.

iii) These multiple variations heavily affect the robustness of the measure of inequality. When one single country is analysed, the range of values that the

[9] The pattern for all the combinations of countries and indices is available from the author upon request.

index might take can be wider than changes in "real" inequality: in Table 4, between the highest and the lowest value that we can obtain for Ireland 1987 using Gini coefficient there is a difference of 12% which is larger than any "real" change of inequality experienced by that country over time. More important, the ranking of countries in the "inequality league" can be affected by the parameter chosen. These findings invoke a very careful reading of those comparative studies of inequality that bring together different definitions of income, recipient units and equivalence scales.

iv) Using two-parameter equivalence scales, an increase in the weight of the adults decreases inequality while an increase in the weight of children increases inequality very heavily. For extreme values of the weights (high α_1 and low α_2) the change in the other weight can provoke a J or an inverted J-shape. These results are not a "law" but stylised facts due to the particular pattern of distribution of adults and children along the distribution of total household income. When the two weights α_1 and α_2 grow together, both a U-shape or a monotonic increase in the measure of inequality can be depicted. The shape of the curve is mainly determined by the index used: indices more sensitive to extreme incomes (Coefficient of Variation and Atkinson(2.0) are more likely to show a monotonic increase in inequality (Figure 16) than indices more sensitive to the central part of the distribution (Gini, Atkinson(0.5) and Theil) show a U-pattern (Figure 15).

v) The two-parameter scales empirically used are very close to the one-parameter scale with a value of around 0.5-0.6. The OECD scale slightly overmeasures inequality compared to one-parameter with $\varepsilon = 0.5$.

vi) Also with respect to two-parameter scales, the robustness of results depends on the index used, the country under examination and the distribution of children and adults within each household. In theory a broad spectrum of heterogeneous results might appear.

vii) Empirically, inequality as measured using the most common equivalence scales does not change considerably. This is not due to an intrinsic robustness of results but to the fact that equivalence scales empirically used have very similar underlying assumptions. On the other side, the use of odd scales (e.g., $\alpha_1 = 0$ and $\alpha_2 = 1$) can produce very particular results. This general conclusion is twinned to another similar conclusion regarding the indices used. The most common indices of inequality provide very similar estimates because they make very similar assumptions regarding the aversion to inequality. Odd aversions to inequality (as in the case of the coefficients of variation or the Atkinson (ε=2) index) produce odd results.[10]

viii) While we need to be aware of the sensitivity of results to changes in the equivalence scale, in empirical studies there is a tendency in using a value for ε of about 0.5. Yet, a comparison of well-being between countries should allow ε to take different values for each country in order to catch in a more precise way the country's peculiarity in terms of household structure and within-household economies of scale. As we can easily figure out, this might have disruptive consequences on the way in which inequality is measured but, without any doubt, further research is needed in this area.

[10] For a study of how inequality changes with respect to the indices used, see Figini (1998).

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Table 1: Inequality Considering Different Recipient Units

Country	Unit	Gini	Theil	Atkinson0.5
AUSTRALIA '89	H	0.354	0.210	0.107
AUSTRALIA '89	Y	0.333	0.191	0.093
BELGIUM '92	H	0.301	0.150	0.081
BELGIUM '92	Y	0.251	0.108	0.057
CANADA '91	H	0.339	0.192	0.097
CANADA '91	Y	0.312	0.167	0.081
CZECH REPUBLIC '92	H	0.297	0.152	0.073
CZECH REPUBLIC '92	Y	0.210	0.086	0.039
DENMARK '92	H	0.342	0.201	0.103
DENMARK '92	Y	0.248	0.120	0.059
FINLAND '91	H	0.313	0.158	0.081
FINLAND '91	Y	0.256	0.114	0.054
FRANCE '89	H	0.390	0.272	0.145
FRANCE '89	Y	0.380	0.263	0.134
HUNGARY '91	H	0.364	0.229	0.115
HUNGARY '91	Y	0.294	0.165	0.081
IRELAND '87	H	0.381	0.252	0.124
IRELAND '87	Y	0.359	0.234	0.111
ISRAEL '92	H	*0.347*	*0.199*	*0.098*
ISRAEL '92	Y	*0.355*	*0.222*	*0.102*
ITALY '91	H	0.330	0.182	0.091
ITALY '91	Y	0.313	0.172	0.082
NETHERLANDS '91	H	0.325	0.191	0.097
NETHERLANDS '91	Y	0.316	0.187	0.091
NORWAY '91	H	0.333	0.189	0.095
NORWAY '91	Y	0.253	0.114	0.055
POLAND '92	H	*0.323*	*0.177*	*0.086*
POLAND '92	Y	*0.326*	*0.184*	*0.088*
RUSSIA '92	H	0.501	0.631	0.230
RUSSIA '92	Y	0.440	0.550	0.187
SLOVAKIA '92	H	0.285	0.135	0.067
SLOVAKIA '92	Y	0.202	0.074	0.035
SPAIN '90	H	0.349	0.211	0.102
SPAIN '90	Y	0.326	0.194	0.091
SWEDEN '92	H	0.329	0.178	0.091
SWEDEN '92	Y	0.251	0.108	0.054
SWITZERLAND '82	H	0.385	0.308	0.137
SWITZERLAND '82	Y	0.361	0.274	0.117
TAIWAN '91	H	0.338	0.203	0.096
TAIWAN '91	Y	0.322	0.194	0.086
UK '91	H	0.389	0.271	0.127
UK '91	Y	0.362	0.245	0.111
USA '91	H	*0.372*	*0.227*	0.117
USA '91	Y	*0.374*	*0.235*	0.117

Notes: H = total household income; Y = household income per-capita. Household inequality measured using HH weights. In italics cases where inequality increases moving from H to Y.

Table 2: Measure of Inequality (Coefficient of Variation) of a Sample Distribution when Different Adjustments for Household Size are Made

Individual label	Individual income when different parameters are considered			
	ε = 0	**ε = 0.3**	**ε = 0.6**	**ε = 1**
1, A	10	10	10	10
2, A	18	14.62	11.88	9
2, B	18	14.62	11.88	9
3, A	26	18.70	13.45	8.67
3, B	26	18.70	13.45	8.67
3, C	26	18.70	13.45	8.67
4, A	35	21.60	13.33	7
4, B	35	21.60	13.33	7
4, C	35	21.60	13.33	7
4, D	35	21.60	13.33	7
4, E	35	21.60	13.33	7
Mean income of the sample	27.181	18.485	12.796	8.092
Coeff. of Var.	**0.324**	**0.210**	**0.086**	**0.137**

Notes: Income distribution adjusted for household size (person weights). Number of households: 4. Total household income: [10, 18, 26, 35]. Household size: [1, 2, 3, 5]. Households are numbered from 1 to 4 while individuals are represented by letters from A to E. In the first column each individual is labelled after his/her belonging to one of the four households. In the following columns adjusted income is computed according to the formula:

$$Y = \text{Total Household Income} / \text{Household size}^{\varepsilon}$$

for values of ε respectively of 0, 0.3, 0.6, 1. Inequality, as measured by the Coefficient of Variation decreases from column 2 to 3 and to 4 because of the equalising effect due to increasing ε. Re-ranking effect, which starts in column 3 (members of household Nr. 4 become poorer than members of household 3), becomes more evident in column 4. Its disequalising effect becomes so strong in column 4 to drive the overall measure of inequality, as the coefficient of variation shows, up again.

Table 3: Values of ε for which Inequality is Minimised for Alternative Choices of Country, Index and Weighting Procedure

	$Gini_{HH}$	**$Gini_{PP}$**	**CV_{HH}**	**CV_{PP}**	**$Theil_{HH}$**	**$Theil_{PP}$**	**Atk_{HH}**	**Atk_{PP}**
Ireland	0.6	0.4/0.5	0.5	0.3	0.5/0.6	0.4/0.5	0.5/0.6	0.4/0.5
Italy	0.5	0.4/0.5	0.4	0.4	0.4/0.5	0.4	0.5	0.4/0.5
UK	0.6/0.7	0.4/0.5	0.4	0	0.6	0.4/0.5	0.6/0.7	0.4/0.5
US	0.5	0.3/0.4	0.4	0.3	0.4/0.5	0.3/0.4	0.5	0.3/0.4/0.5

Notes: PP = person weights; HH = household weights.

Table 4 - Inequality Ranking Using Different Assumptions

	Gini$_{PP}$, ε = 0.5	**Gini$_{HH}$, ε = 1**
UK	0.341 (1)	0.363 (3)
US	0.337 (2)	0.364 (2)
Ireland	0.330 (3)	0.375 (1)

Notes: in column 2, Gini is computed using PP weights and ε is set equal to 0.5. In column 3, Gini is computed using HH weights and ε is set equal to 1.

Table 5: Household Size and Composition in Selected Countries

	Ireland (1987)			Italy (1991)			United Kingdom			United States (1991)		
Deciles	size	adults	child.	size	adults	child.	size	adults	child.	size	adults	child.
1	2.83	1.74	1.09	2.61	2.02	.59	1.84	1.39	.45	2.46	1.51	.95
2	3.70	2.03	1.67	3.21	2.28	.93	2.42	1.68	.74	2.97	1.82	1.15
3	4.74	2.15	2.59	3.44	2.41	1.03	2.88	1.84	1.04	3.31	1.98	1.33
4	4.74	2.28	2.46	3.54	2.55	.99	3.14	1.97	1.17	3.33	2.09	1.24
5	4.67	2.39	2.28	3.61	2.68	.93	3.24	2.07	1.17	3.61	2.23	1.38
6	4.87	2.64	2.23	3.69	2.79	.90	3.39	2.19	1.20	3.59	2.24	1.35
7	5.15	2.83	2.32	3.75	2.89	.86	3.55	2.31	1.24	3.56	2.36	1.20
8	5.14	2.85	2.29	3.91	3.03	.88	3.42	2.40	1.02	3.66	2.46	1.20
9	5.36	3.37	1.99	4.13	3.29	.84	3.36	2.47	.89	3.80	2.61	1.19
10	5.49	3.90	1.59	4.13	3.48	.65	3.73	2.72	1.01	4.08	2.93	1.15
CV	0.175	0.249	0.227	0.126	0.165	0.162	0.186	0.189	0.246	0.132	0.182	0.102

Notes: average size, number of adults and number of children in each decile of the population ranked by total household income. A measure of inequality CV (coefficient of variation) for the variables is calculated in the last row.

Table 6: Comparison of Inequality Using Different Equivalence Scales

	One parameter equivalence scale			Two parameter equivalence scale		
	$\varepsilon = 0$	$\varepsilon = 0.5$	$\varepsilon = 1.$	α_1=.7 α_2=.5	α_1=.6 α_2=.4	α_1=.5 α_2=.3
Ireland						
CV	0.730	0.716	0.815	0.735	0.721	0.705
Theil	0.207	0.193	0.234	0.198	0.192	0.185
Atk 0.5	0.103	0.094	0.111	0.095	0.092	0.090
Atk 2.0	0.348	0.339	0.399	0.351	0.342	0.332
Gini	0.345	0.330	0.359	0.331	0.326	0.321
Italy						
CV	0.605	0.586	0.679	0.611	0.597	0.584
Theil	0.158	0.144	0.172	0.149	0.145	0.141
Atk 0.5	0.078	0.071	0.082	0.073	0.071	0.069
Atk 2.0	0.268	0.256	0.316	0.272	0.263	0.254
Gini	0.307	0.291	0.313	0.293	0.290	0.287
UK						
CV	0.841	0.871	0.973	0.901	0.886	0.869
Theil	0.229	0.214	0.245	0.216	0.212	0.208
Atk 0.5	0.109	0.099	0.111	0.099	0.097	0.096
Atk 2.0	0.414	0.432	0.486	0.448	0.440	0.430
Gini	0.357	0.341	0.362	0.340	0.337	0.334
US						
CV	0.643	0.638	0.742	0.666	0.651	0.636
Theil	0.195	0.188	0.235	0.198	0.192	0.185
Atk 0.5	0.101	0.096	0.117	0.101	0.098	0.095
Atk 2.0	0.293	0.290	0.355	0.307	0.298	0.288
Gini	0.345	0.337	0.374	0.346	0.340	0.335

Notes: PP weights have been used in all the computations.

Fig. 1: Gini index, the UK 1991, for both person and household w eighting

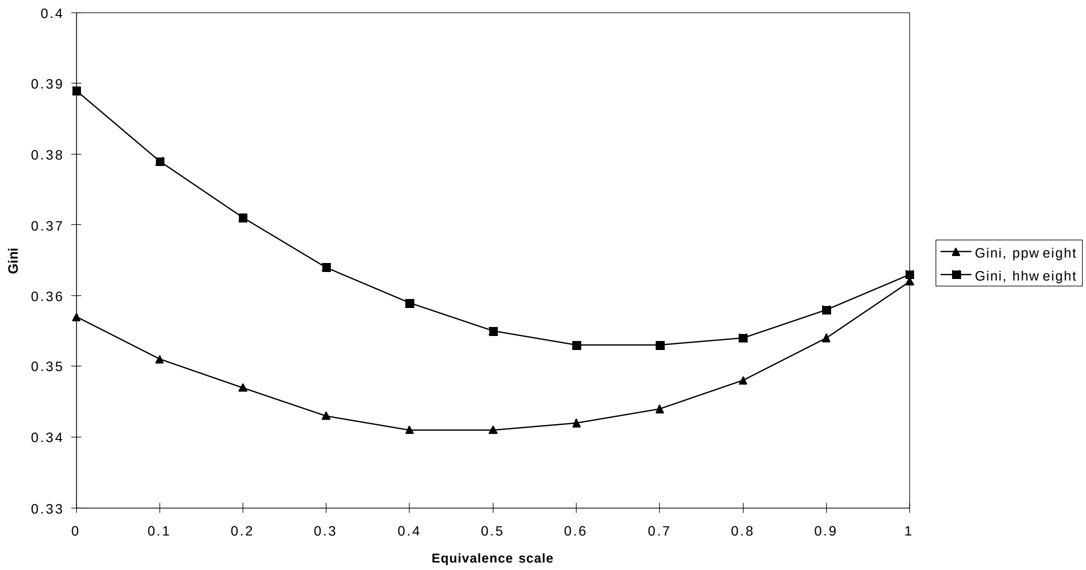

Fig. 2: Coefficient of Variation, the UK 1991, for both person and household w eighting

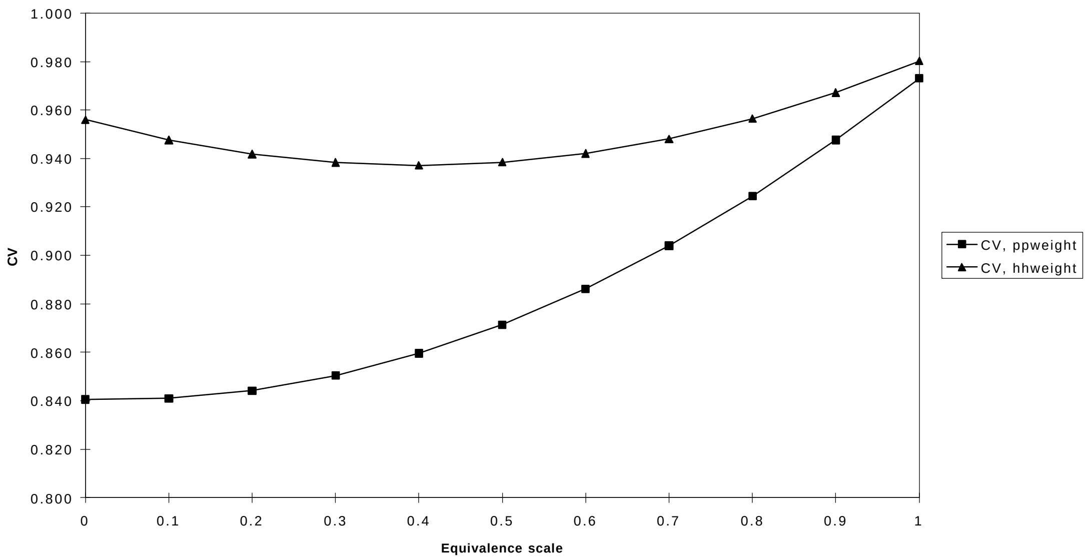

Fig. 3: Theil index, US 1991, Person and household weights

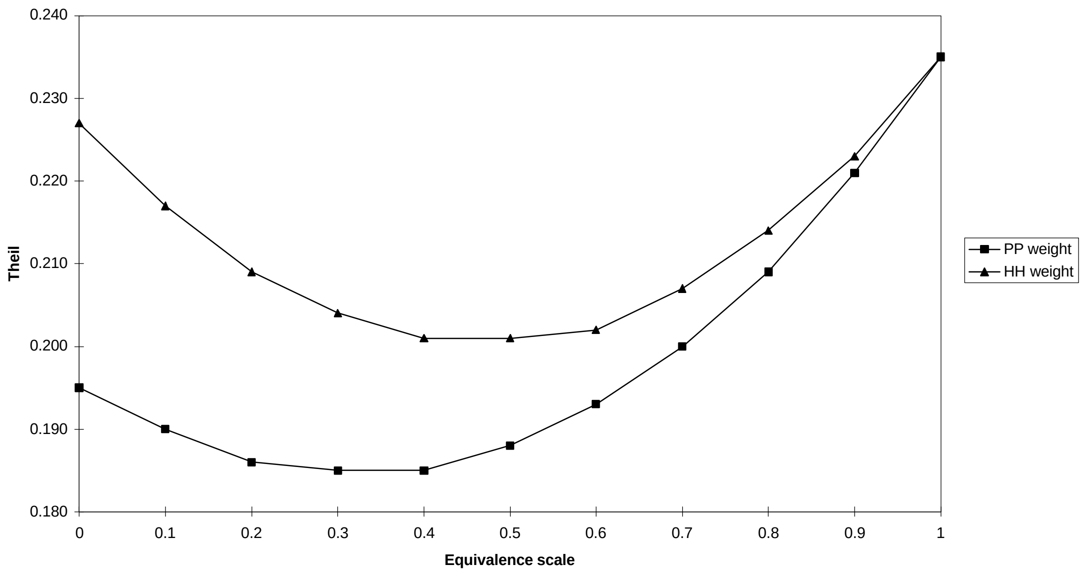

Fig. 4: Atkinson index, US 1991, person and household weights

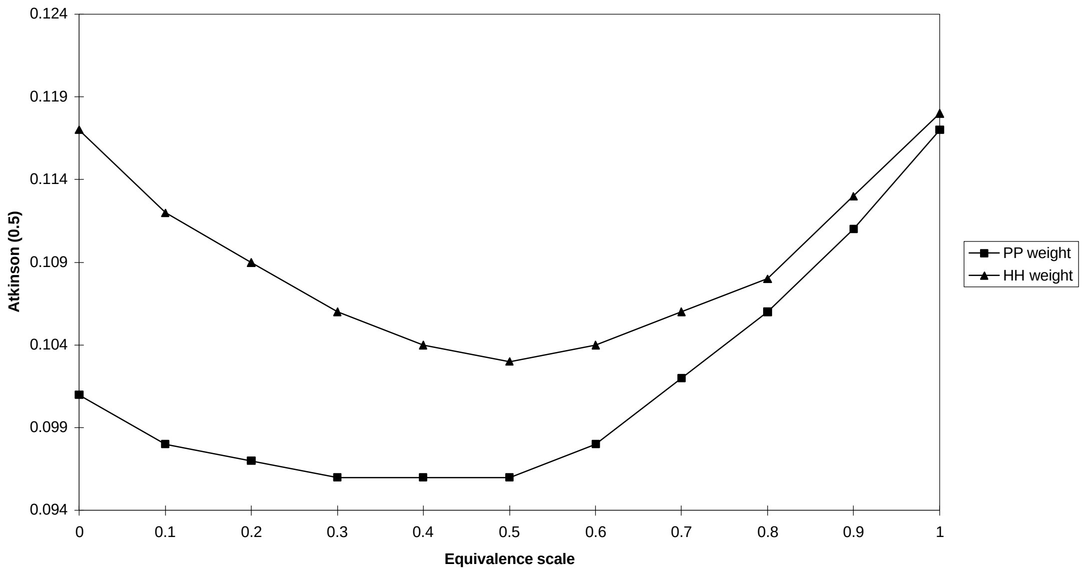

Fig. 5: Gini index, Ireland 1987, person and household w eights

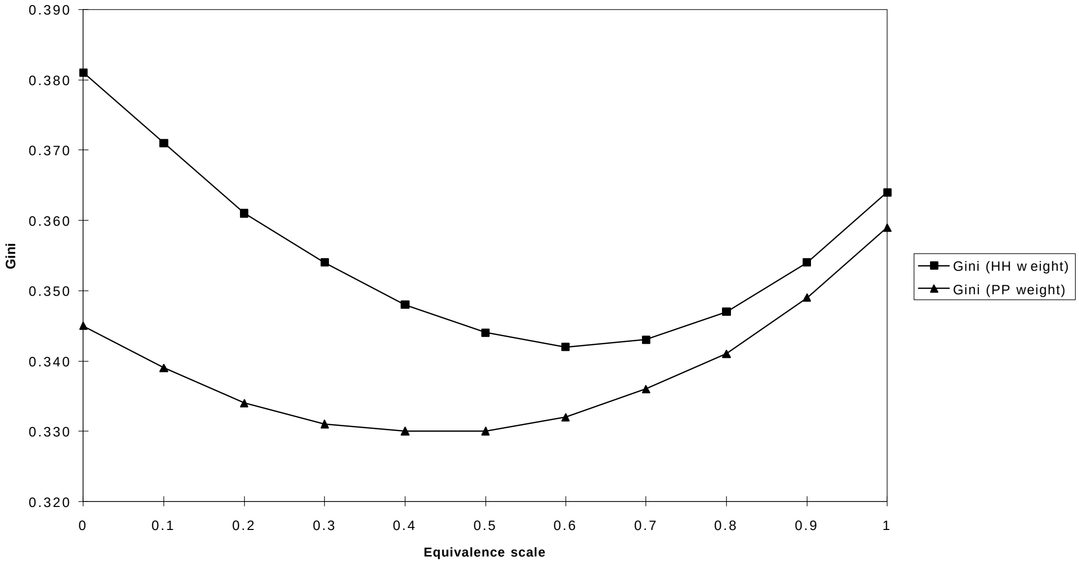

Fig. 6: Atkinson index, Ireland 1987, person and household w eights

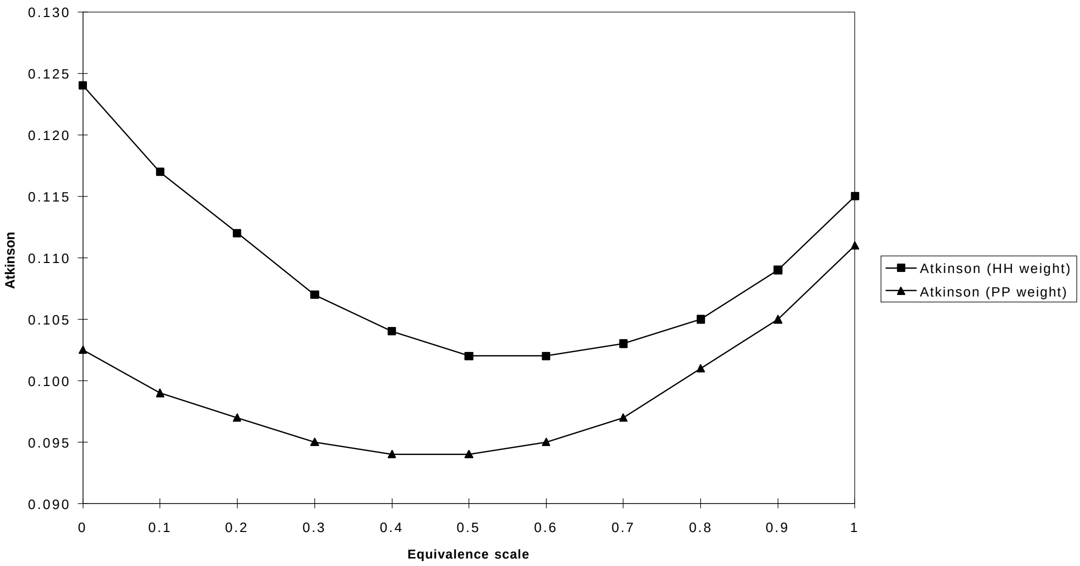

Fig. 7: Coefficient of Variation, Italy 1991, person and household weights

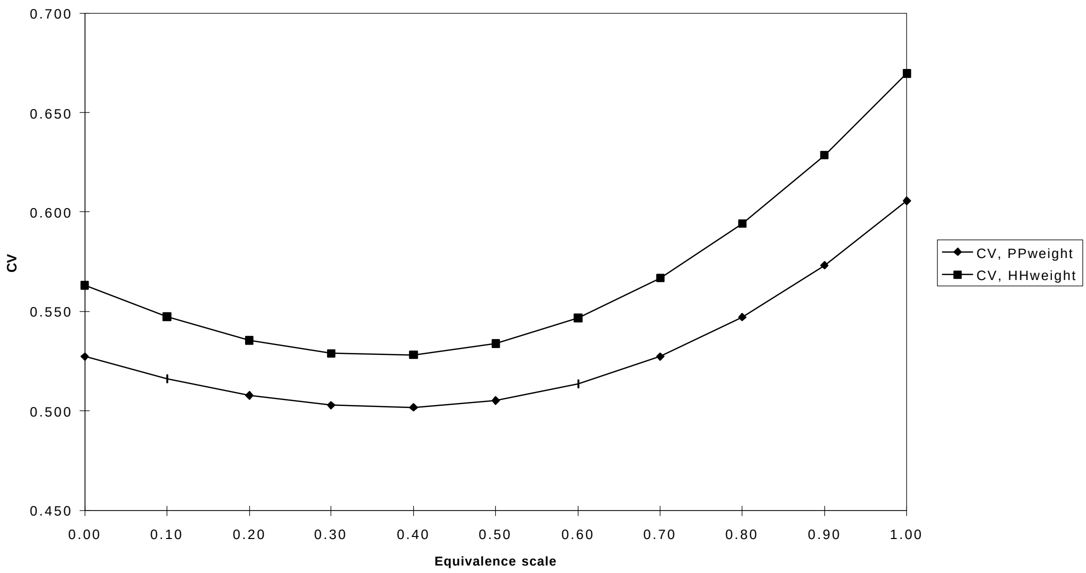

Fig. 8: Atkinson index, Italy 1991, person and household weights

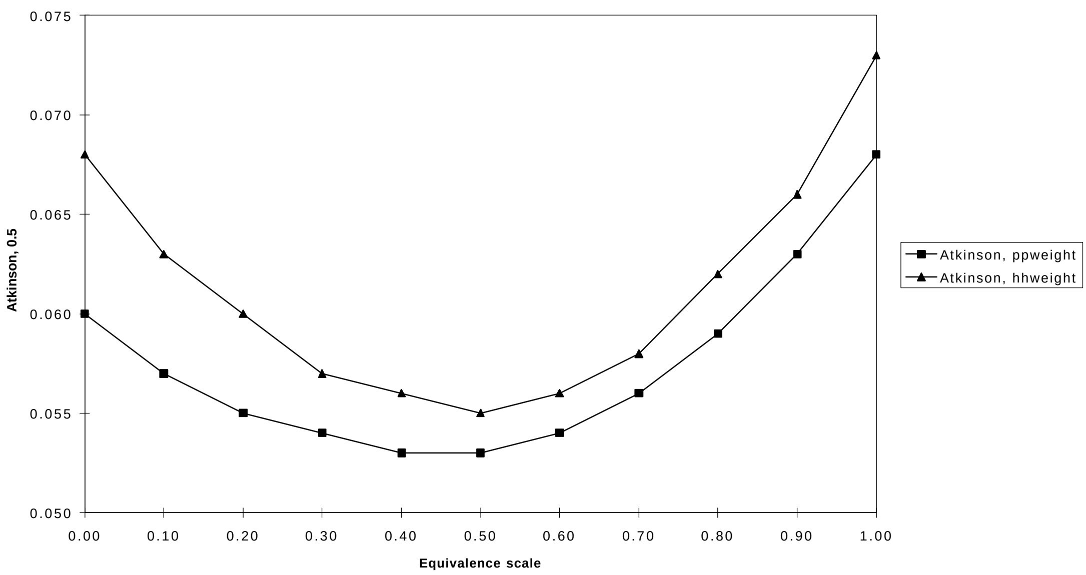

Fig. 9: A Comparison of Inequality for Different Values of the Parameter ε: Ireland, UK and US.

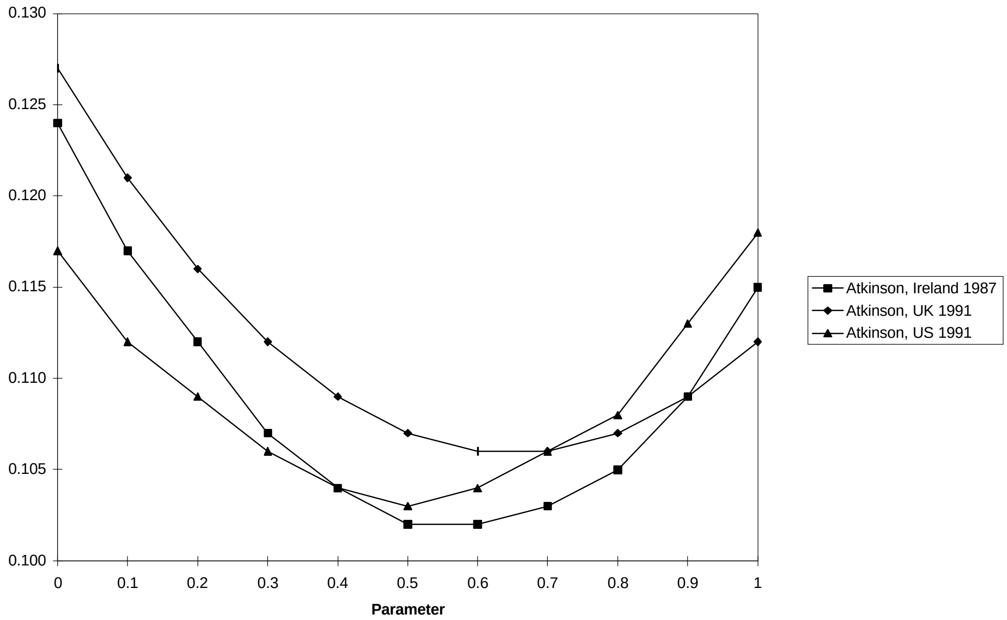

Figure 10 - Measure of Inequality varying the value of α_1 for $\alpha_2 = 0$ (Fig. a) and $\alpha_2 = 1$ (Fig. b)

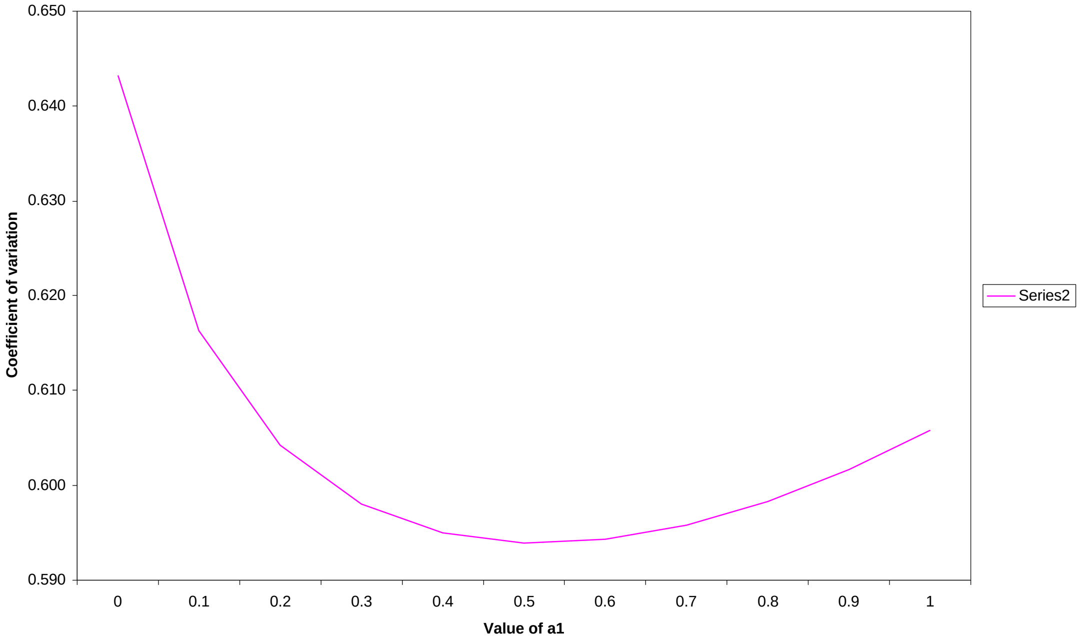

Figure a: Coefficient of Variation, US 1991, PP weights, value of $\alpha_2 = 0$. Figure b: Theil index, US 1991, PP weights, value of $\alpha_2 = 1$.

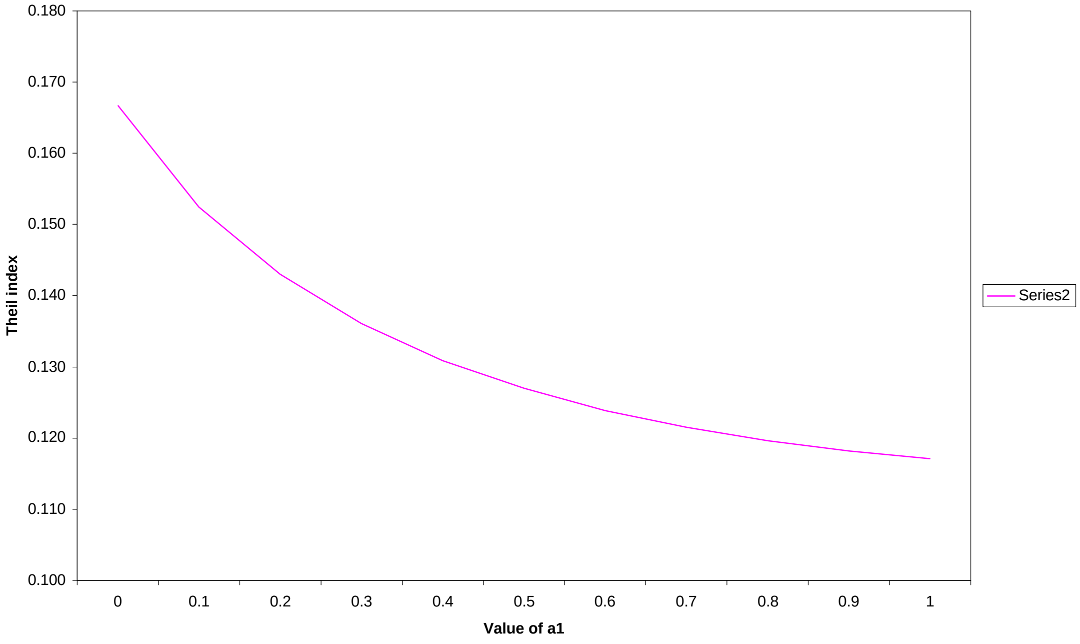

Figure 11 - Measure of Inequality varying the value of α_2 for α_1 = 0 (Fig. a) and α_1 = 1 (Fig. b)

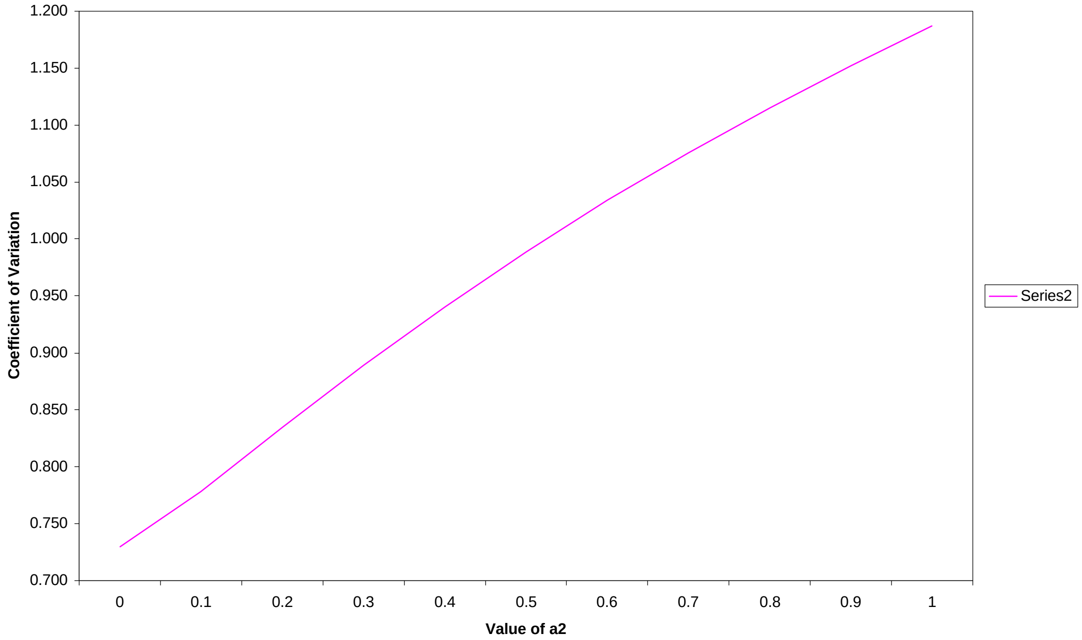

Figure a: Ireland 1987, Coefficient of Variation, PP weights, α_1 = 0. Figure b: Ireland 1987, Theil index, PP weights, α_1 = 1.

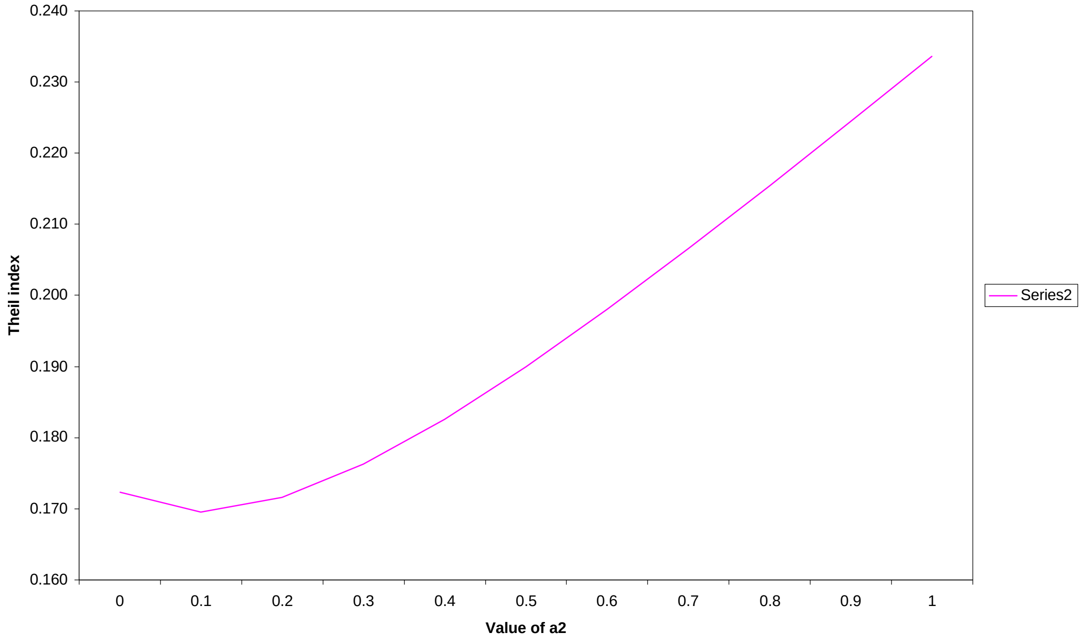

Fig. 12: Theil index, Ireland 1987, two-parameter equivalence scale

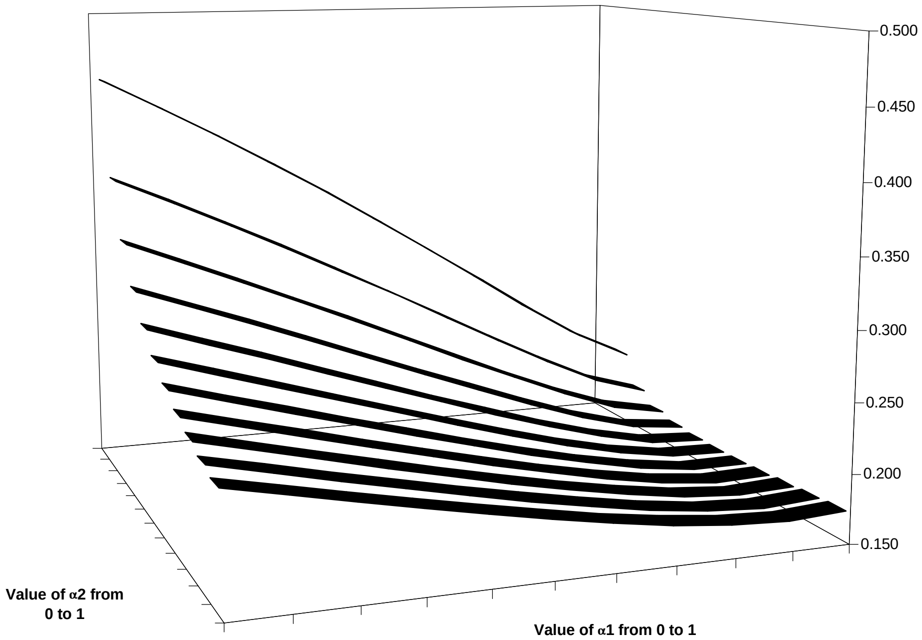

Fig. 13: Coeff. of variation, Ireland 1987, two-parameter equivalence scale

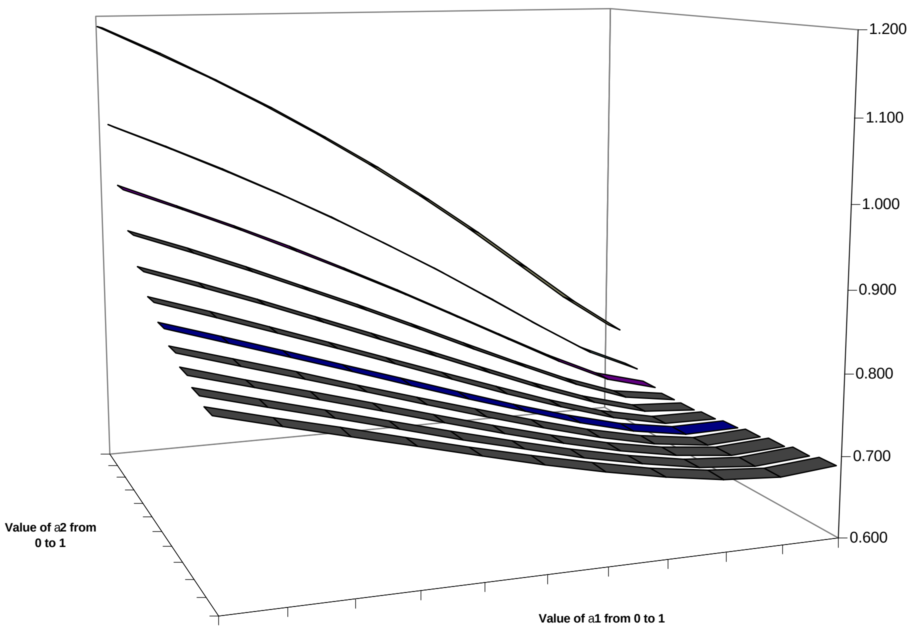

Fig. 14: Atkinson (0.5), US 1991, two-parameter equivalence scale

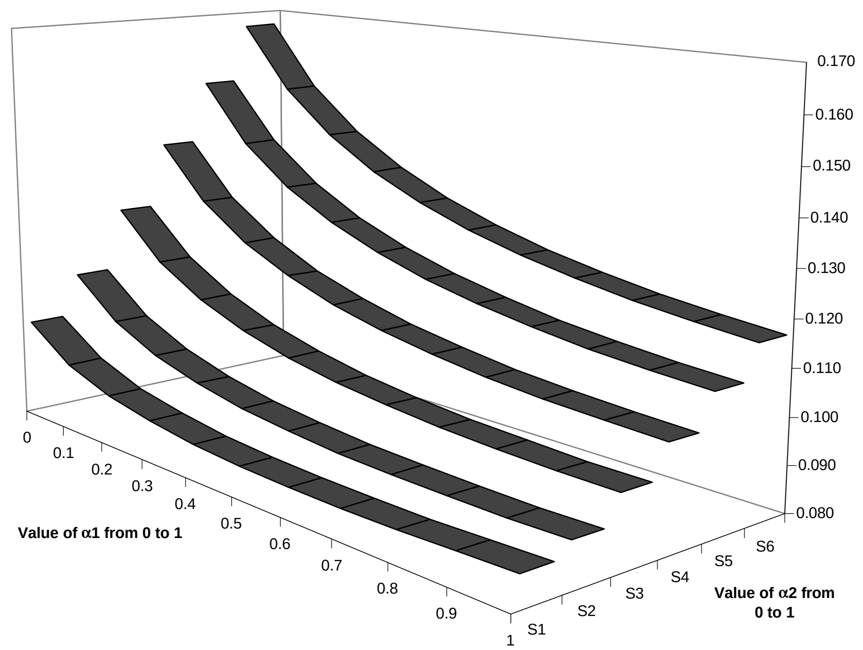

Fig. 15: Atkinson (0.5), UK 1991 when α_1 and α_2 vary together

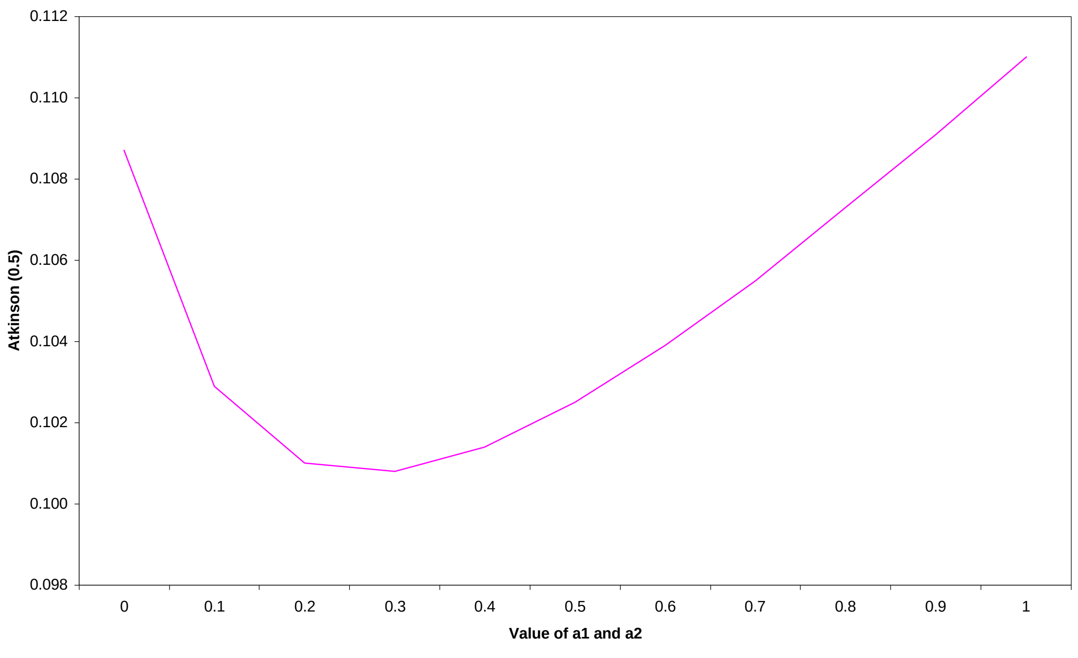

Fig. 16: Atkinson (0.5), UK 1991 when α_1 and α_2 vary together